

Tunneling of correlated electrons in ultra high magnetic field

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Effects of the electron-electron interaction on tunneling into a metal in ultra-high magnetic field (ultra-quantum limit) are studied. The range of the interaction is found to have a decisive effect both on the nature of the field-induced instability of the ground state and on the properties of the system at energies above the corresponding gap. For a short-range repulsive interaction, tunneling is dominated by the renormalization of the coupling constant, which leads eventually to the charge-density wave instability. For a long-range interaction, there exists an intermediate energy range in which the conductance obeys a power-law scaling form, similar to that of a 1D Luttinger liquid. The exponent is magnetic-field dependent, and more surprisingly, may be positive or negative, *i. e.*, interactions may either suppress or enhance the tunneling conductance compared to its non-interacting value. At energies near the gap, scaling breaks down and tunneling is again dominated by the instability, which in this case is an (anisotropic) Wigner crystal instability.

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Low-dimensional systems exhibit “zero-bias anomalies” in tunneling (non-linearities of the current-voltage characteristics at small biases), which reflect the renormalization of the density of states by the electron-electron interaction. In particular, tunneling into a one-dimensional (1D) metal (Luttinger liquid) is characterized by a power-law suppression of the tunneling conductance, g_T . A three-dimensional metal placed in a strong magnetic field that depopulates all but one Landau levels (ultra-quantum limit, UQL) provides an example of a very special quasi-1D system. It is well-known^{1–3} that repulsive interactions in the UQL lead to a charge-density wave (CDW) or a Wigner crystal instability of the ground state. This has been confirmed, for example, by experiments on graphite in high magnetic fields⁴. The most complete analysis of this instability for the case of a short-range interaction was performed in Ref. [3] by solving the renormalization-group (RG) equation for the interaction vertex. On the other hand, it has recently been shown that for the case of a long-range (Coulomb) interaction g_T exhibits a power-law, Luttinger-liquid-like behavior at sufficiently high energies⁵. This result was obtained in two ways – via a perturbative Hartree-Fock procedure, which results in the RG equation for the transmission amplitude, and via bosonization in the coherent state basis – neither of which took into account the renormalization of the interaction vertex. In this paper, we combine the approaches of Refs. [1,3] and [5] to study the behavior of the tunneling conductance in the whole energy interval from the Fermi energy down to the energy associated with the instability. This is accomplished by solving a coupled system of the RG equations for both the interaction vertex and transmission amplitude. We find that for a long-range interaction there is a parametrically wide energy interval in which the flow of the transmission amplitude already results in its power-

law scaling with energy, whereas the renormalization of the vertex is not yet important. In this interval, the system behaves as a Luttinger liquid and the results of Ref. [5] are applicable. At energies close to the gap, the power-law scaling breaks down and tunneling is dominated by the electron properties in the vicinity of the critical point. For a short-range interaction, the critical point dominates tunneling at all energies, and there is no Luttinger-liquid behavior. We also find that above a certain magnetic field, when the lowest Landau level is strongly depopulated, the interaction *enhances* the tunneling conductance of the Luttinger-liquid regime above its free-electron value. Such an unusual behavior receives a natural explanation in terms of electron scattering from the Friedel oscillation near the tunneling barrier.

First we review the procedure of finding the effect of the electron-electron interaction on the tunneling conductance of a metal in the UQL⁵. Let the magnetic field be perpendicular to the contact plane ($z = 0$) that separates two metallic sides. We consider both the symmetric configuration, where both sides are in the UQL, and the asymmetric one, where one of the sides is made of a high-carrier concentration and/or dirty metal so that, in the first approximation, the magnetic field does not affect that side. The transmission and reflection amplitudes for non-interacting electrons, t_0 and r_0 , are assumed to be known. We choose the Landau basis for the free electron wavefunctions (non-interacting electrons in the UQL and in the presence of the barrier):

$$\psi_{p_z, p_x}^{(0)}(\mathbf{r}) = \psi_{p_z}(z) \chi_{p_x}(x, y),$$

where

$$\begin{aligned} \psi_{p_z}(z) &= \theta(z) t_0 e^{ip_z z} + \theta(-z) (e^{ip_z z} + r_0 e^{-ip_z z}) \\ \psi_{-p_z}(z) &= \theta(z) (e^{-ip_z z} + r_0 e^{ip_z z}) + \theta(-z) t_0 e^{-ip_z z}. \end{aligned} \quad (1)$$

(Here and thereafter, we set $\hbar = 1$ and also define the magnetic length as our unit of length: $\sqrt{\hbar c/eB} \equiv 1$).

Backscattering of magnetically quantized electrons at the barrier gives rise to a Friedel oscillation in the charge density, whose amplitude decays away from the barrier as z^{-1} . Correspondingly, the exchange, $V_{ex}(\mathbf{r}, \mathbf{r}')$, and Hartree, $V_H(\mathbf{r})$, potentials calculated using exact (in the presence of the barrier) but otherwise free wavefunctions, also exhibit rapid $2k_F$ -oscillations and decay as z^{-1} away from the barrier. The first-order correction to the wavefunction due to interaction is given by

$$\delta\Psi_{p_z, p_x}(\mathbf{r}) = \int d\mathbf{r}' \int d\mathbf{r}'' G(\mathbf{r}, \mathbf{r}'; E) [V_{ex}(\mathbf{r}', \mathbf{r}'') + \delta(\mathbf{r}' - \mathbf{r}'') V_H(\mathbf{r}'')] \Psi_{p_z, p_x}^{(0)}(\mathbf{r}''), \quad (2)$$

where $G(\mathbf{r}, \mathbf{r}'; E)$ is the Green's function in the presence of the magnetic field. When the momentum transfer ($2p_z$) matches the wavevector of the Friedel oscillation ($2k_F$), a slow (z^{-1}) decay of the oscillations in V_{ex} and V_H leads to a logarithmic singularity in $\delta\Psi(\mathbf{r})$, and consequently, in δt . For electron's energy close to the Fermi energy, we have

$$t = t_0 [1 - |r_0|^2 c \Gamma_0 \ln(W/E)] \quad (3)$$

where $c = 1(1/2)$ for symmetric (asymmetric) configuration, $W \sim E_F$ is the effective bandwidth, $\Gamma_0 \equiv \Gamma(\mathbf{q}_\perp = 0)$, and $\Gamma(\mathbf{q}_\perp)$ is expressed via the Fourier transform of the interaction potential $U_0(q_z, \mathbf{q}_\perp)$:

$$\Gamma(\mathbf{q}_\perp) = \frac{1}{(2\pi)^2 v_F} \left[\int \frac{d^2 k_\perp}{2\pi} e^{i\mathbf{q}_\perp \cdot \mathbf{k}_\perp - k_\perp^2/2} U_0(0, \mathbf{k}_\perp) - U_0(2k_F, \mathbf{q}_\perp) e^{-q_\perp^2/2} \right]. \quad (4)$$

The interaction potential, U_0 , can be both of long- and short-range. The first situation corresponds to a single-band metal, in which there are no more charge carriers besides those already in the UQL. In this case, $U_0(q_z, \mathbf{q}_\perp) = 4\pi e^2/(q^2 + \kappa^2)$ (for $q \ll k_F$), where $\kappa = \omega_p/v_F$ and ω_p is the plasma frequency at $B = 0$. The perturbation theory works if $e^2/v_F \simeq \kappa^2 \ll 1$, which means that the interaction is long range. As the magnetic field reduces the phase-space available for the motion along the field, k_F and v_F decrease with the field (in the UQL, $k_F, v_F \propto B^{-1}$). Therefore, the perturbation theory breaks down at sufficiently strong field so that $\kappa \simeq 1$. The case of a short-range interaction may correspond to a situation when a small pocket of the Fermi surface is in the UQL, whereas other parts of the Fermi surface still contain many Landau levels. Screening of the interaction among the magnetically quantized carriers is then mostly due to the “external” carriers, and the interaction may be of short-range.

The first and second terms in Eq.(4) are the exchange and Hartree contributions, respectively. Due to the Pauli principle, they enter Eq.(4) with opposite signs. Note

that exchange contribution involves integration over the transverse momentum \mathbf{k}_\perp , whereas the Hartree contribution enters Eq.(4) at $q_\perp = 0$. This is due to a simple fact that the Friedel oscillation in charge density, and hence in the Hartree potential, is translationally-invariant along the barrier, whereas the exchange potential involves the density matrix and is thus non-local. If the interaction potential is peaked strongly at $q = 0$ (e.g., Coulomb potential) the Hartree contribution is enhanced due to $q_\perp = 0$ and may in fact *dominate* over the exchange one. In this case Γ_0 in Eq.(3) is negative and thus the transmission amplitude is *enhanced* by the interaction. Such an unusual behavior should be contrasted, e.g., to a strictly 1D case⁶, in which $\Gamma_0 = U_0(0) - U_0(2k_F)$ and is thus positive for any “realistic” interaction potential, i.e., such that $U_0(q)$ is a monotonically decreasing function of q . In our case, due to the 3D nature of the problem, the exchange and Hartree contributions are not simply given by the corresponding forward and backscattering amplitudes but also involve averaging over the transverse direction. As a result, Γ_0 can take either sign.

At the next order, V_{ex} and V_H are re-calculated using the corrected wavefunctions and are substituted back into Eq.(2). The higher order corrections to $\delta\Psi(\mathbf{r})$ are higher powers of logs. Similarly to tunneling through a barrier of weakly interacting electrons in 1D,⁶ summation of the most divergent corrections to t in all orders of the perturbation theory can be performed via an RG equation

$$\frac{dt}{d\xi} = -c\Gamma_0 t (1 - |t|^2), \quad (5)$$

where $\xi \equiv \ln(W/E)$ and $t(\xi = 0) = t_0$. The solution of (5) is

$$t = \frac{t_0(E/W)^{c\Gamma_0}}{\sqrt{|r_0|^2 + |t_0|^2(E/W)^{c\Gamma_0}}}. \quad (6)$$

For small values of $|t_0|$ (high barrier), Eq.(6) reduces to a power-law form $t = t_0 (E/W)^{c\Gamma_0}$. The tunneling conductance is related to the transmission amplitude via the Landauer formula.

The power-law solution for t is obtained using the bare interaction vertex. However, the vertex is also subject to renormalization. The flow of $\Gamma(\mathbf{q}_\perp, \xi)$ is described by the integro-differential RG equation³, which has a more transparent meaning when written for the Fourier transform of $\gamma(\mathbf{r}_\perp, \xi) = 1/(2\pi)^2 \int d^2 q_\perp \Gamma(\mathbf{q}_\perp, \xi) e^{-i\mathbf{q}_\perp \cdot \mathbf{r}_\perp}$:

$$\frac{d\gamma(\mathbf{r}_\perp, \xi)}{d\xi} = \int d\mathbf{r}'_\perp \gamma(\mathbf{r}'_\perp, \xi) \gamma(\mathbf{r}_\perp - \mathbf{r}'_\perp, \xi) \times \left(1 - e^{i\mathbf{r}_\perp \wedge \mathbf{r}'_\perp}\right), \quad (7)$$

where $\mathbf{a} \wedge \mathbf{b} = a_x b_y - a_y b_x$. The initial condition for Eq. (7) is given by $\Gamma(\mathbf{q}_\perp, \xi = 0) = \Gamma(\mathbf{q}_\perp)$, where $\Gamma(\mathbf{q}_\perp)$ is defined by Eq. (4). The flow of the vertex affects the RG equation for t , in which now the bare vertex has to be replaced by the renormalized one:

$$\frac{dt}{d\xi} = -c\Gamma(\mathbf{q}_\perp = 0, \xi) t (1 - |t|^2). \quad (8)$$

Eqs. (4), (7) and (8) provide a full description of tunneling into a three-dimensional metal in the UQL.

Few words about the general features of Eq.(7) are now in place. Physically, $\gamma(\mathbf{r}_\perp, \xi)$ plays the role of an interaction potential between two electrons whose guiding centers are separated by a “distance” $|\mathbf{r}_\perp|$ across the magnetic field. (Note however that $\gamma(\mathbf{r}_\perp, 0)$ is related to the Fourier transform of the bare interaction, cf. Eq. (4).) The first and second terms in the brackets in Eq. (7) correspond to the Peierls and Cooper interaction channels, respectively. A strictly 1D case is recovered by putting all guiding centers onto the same line: $\mathbf{r}_\perp = \mathbf{r}'_\perp$. In this case, $\mathbf{r}_\perp \wedge \mathbf{r}'_\perp = 0$ and the Peierls and Cooper channels cancel each other exactly. This cancellation reflects the well-known fact that there are no instabilities for 1D (spinless) electrons and the system remains in the massless phase (Luttinger liquid) down to the lowest energies. In the UQL case $\mathbf{r}_\perp \neq \mathbf{r}'_\perp$, Peierls and Cooper channels do not cancel each other, and an instability becomes possible.

For a short-range interaction, the q_\perp -dependence of U_0 can be neglected: $U_0(0, \mathbf{q}_\perp) \rightarrow u_0$, $U_0(2k_F, \mathbf{q}_\perp) \rightarrow u_{2k_F}$ and $\Gamma(\mathbf{q}_\perp) = (u_0 - u_{2k_F}) / (2\pi)^2 v_F e^{-q_\perp^2/2}$. For repulsion, the solution of Eq. (7) with the Peierls term alone is^{1,3}

$$\Gamma(\mathbf{q}_\perp, \xi) = (\Gamma^{-1}(\mathbf{q}_\perp) - \xi)^{-1}. \quad (9)$$

The vertex diverges most rapidly at $\mathbf{q}_\perp = 0$ and the equation for the pole $\xi = \Gamma^{-1}(0)$ yields the gap $\Delta = W \exp(-1/\Gamma_0)$. As the electron density remains homogeneous across the field, the instability correspond to a CDW along the field. Inclusion of the Cooper term merely changes the magnitude of the gap³. The RG equation for $t(\xi)$ [Eq.(8)] with $\Gamma(\mathbf{q}_\perp, \xi)$ from Eq.(9) allows for an analytic solution:

$$t = \frac{t_0 f(E)}{\sqrt{|r_0|^2 + |t_0|^2 f^2(E)}}, \quad (10)$$

where $f(E) \equiv [\Gamma_0 \theta(E - \Delta) \ln E / \Delta]^c$. Near the gap ($E \approx \Delta$), the tunneling conductance $g_T \propto |t|^2$ assumes a *universal* form

$$g_T \propto (E - \Delta)^{2c} \theta(E - \Delta). \quad (11)$$

Comparing t from Eq.(10) with Eq. (6), obtained without taking into account the flow of the interaction vertex Γ , we see that the two coincide only at the lowest order in the interaction (first log-correction to t_0). In contrast to Eq.(6), the full solution shows that the higher order logarithmic terms do not sum up into a power-law. This is already seen from the fact that the first log-correction to t_0 [Eq.(3)] becomes of order t_0 at $E \simeq \Delta$, *i.e.*, when the renormalization of Γ cannot be neglected. In other words, the CDW critical point dominates the physical properties in the entire energy interval from W to Δ .

We now turn to the case of a long-range interaction. It is instructive to consider first an example of an ultra-long interaction: $U_0(\mathbf{r}) = \text{const} \rightarrow \gamma(\mathbf{r}_\perp, 0) \propto \delta(\mathbf{r}_\perp)$. To analyze the flow of $\gamma(\mathbf{r}_\perp, \xi)$ for small ξ it suffices to substitute $\gamma(\mathbf{r}_\perp, 0)$ into the right-hand side of Eq.(7), upon which the latter vanishes. It means that the ultra-long range interaction is equivalent to a strictly 1D case, when $\gamma(\mathbf{r}_\perp, \xi)$ is invariant under renormalization. The example just given is certainly not realistic: even for the Coulomb interaction the ground state is unstable with regard to the CDW formation². However, remnants of this behavior survive for the Coulomb potential in a sense that $\gamma(\mathbf{r}_\perp, \xi)$ flows *slower* than $t(\xi)$. As a result, there exists an intermediate energy scale, E_{PL} , ($\Delta \ll E_{PL} \ll W$) such that at $E \simeq E_{PL}$ the higher order logarithmic terms generated by Eq.(8) have already summed up into a power-law form but the renormalization of Γ is not significant yet. E_{PL} may be defined by the condition that the lowest-order correction to t_0 becomes equal to t_0 :

$$\xi_{PL} = \ln(W/E_{PL}) = 1/c\Gamma_0.$$

For a screened Coulomb potential, Eq.(4) together with the condition $\kappa \ll k_F \lesssim 1$ yields

$$\Gamma_0 = (e^2/\pi v_F) \left[|\ln \kappa| - (4k_F^2)^{-1} \right]. \quad (12)$$

A simple estimate for E_{PL} can be obtained if the first (exchange) term in Eq.(12) is larger than the second (Hartree) one, in which case $E_{PL} \simeq W \exp(-\pi v_F / c e^2 |\ln \kappa|)$. At the same time, for $k_F \simeq 1$, the gap is estimated as $\Delta = W \exp(-a v_F / e^2)$, where $a \simeq 1$. We see that $E_{PL} \gg \Delta$ due to the presence of a large logarithm under the exponential.

As it has already been pointed out, Γ_0 is *not* necessarily positive: if k_F is sufficiently small (the lowest Landau level is strongly depleted), the Hartree (backscattering) term dominates and $\Gamma_0 < 0$. The dependence of Γ_0 on k_F is shown in the inset of Fig. 1 for a range of κ . Recalling that both k_F and κ are magnetic-field dependent, one can express the field B_0 at which Γ_0 changes sign in terms of the nominal field B_Q , at which the UQL is achieved, and the familiar gas parameter, r_s , as $B_0 \approx 2B_Q / \ln^{1/3} C r_s^{-1}$, where $C \simeq 1$. For $r_s \simeq 1$ $B_0 \approx 2B_Q$. The gap as a function of k_F and κ was obtained by solving the RG equation (7) numerically with Eq.(4) as an initial condition. The ratio $\xi_c/\xi_{PL} \equiv (\ln W/\Delta)/\ln(W/E_{PL})$ is plotted in Fig. 1. All numerical results presented here are for a symmetric contact, *i. e.*, $c = 1$. For a sufficiently long-range interaction (cf. filled circles for $\kappa = 0.1$ in Fig. 1) there is a wide range of k_F (and thus of the magnetic field) in which $\xi_c/\xi_{PL} > 1$. Once this condition is satisfied, there exists an energy interval $\Delta \ll E \lesssim E_{PL}$ ($\xi_{PL} \lesssim \xi \lesssim \xi_c$) in which the system effectively behaves as a 1D Luttinger liquid, exhibiting a characteristic power-law scaling of the tunneling conductance. (For $E_{PL} \ll E \ll W$ the power-law reduces to the first log-correction.) Near the gap ($E \simeq \Delta$), the power-law behavior crosses over to the threshold behavior [Eq. (11)]. If the interaction is

not sufficiently long-ranged, there is no energy interval in which $\xi_c/\xi_{PL} > 1$ is satisfied. This is illustrated by the open circles in Fig. 1 ($\kappa = 0.6$). (We did not plot the points corresponding to $k_F < \kappa$, as it would have been inconsistent with our weak-coupling approximation.)

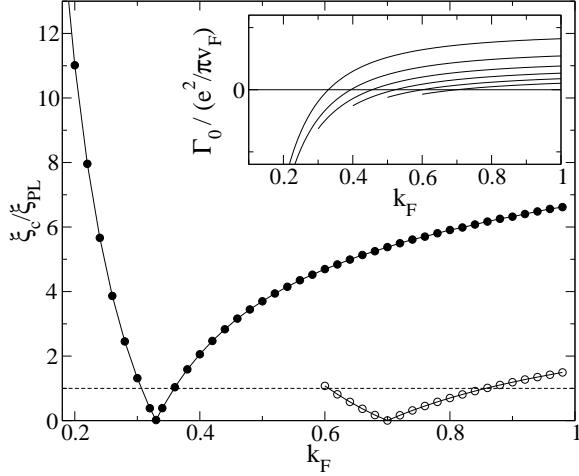


FIG. 1. Ratio ξ_c/ξ_{PL} for $\kappa = 0.1$ (filled circles) and $\kappa = 0.6$ (open circles). The dashed line corresponds to the value 1. Inset: $\Gamma_0/(e^2/\pi v_F)$ as function of k_F for various values of κ (from top to bottom: $\kappa = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$).

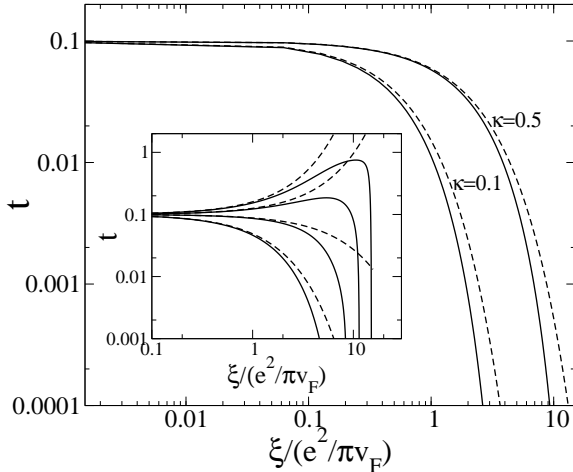


FIG. 2. Solid lines: solution of the full system of RG equations Eqs. (8, 7, 4) for the screened Coulomb potential. Dashed lines: solutions of Eq. (8) with Γ_0 given by its bare value. In the main plot $t_0 = 0.1$, $k_F = 0.8$ and the values of κ are indicated in the figure. Inset: $k_F = 0.4$. The curves, from bottom to top, correspond to $\kappa = 0.1, 0.2, 0.3$ and 0.4 .

A full solution of Eqs. (4,7,8) illustrating various regimes of the $t(E)$ -dependence for a screened Coulomb potential is represented by solid lines in Fig. 2 for $t_0 = 0.1$. Dashed lines are solutions of Eq. (8) only with Γ_0 given by its bare value [Eq. 4], *i. e.*, without taking into account the renormalization of the coupling constant. In the main panel, $k_F = 0.8$. The region of small ξ shows

a good agreement between the full solution (solid lines) and the Luttinger-liquid solution (dashed lines). As E approaches Δ ($\xi \rightarrow \xi_c$), scaling breaks down and $t(E)$ vanishes at $E = \Delta$. In the inset of Fig. 2, $k_F = 0.4$. The rise of t with ξ corresponds to the negative value of the bare vertex, Γ_0 . This corresponds to an enhancement of g_T above its non-interacting value. Had the vertex renormalization been absent, t would have reached unity in the limit $E \rightarrow 0$ ($\xi \rightarrow \infty$). However, the vertex renormalization curbs the rise in t and eventually brings it down to zero at $E = \Delta$. At the instability, the system becomes an insulator. For long-range interaction, we find that the vertex diverges most rapidly at finite q_\perp (for $\kappa \rightarrow 0$, $q_\perp \rightarrow \approx 0.7$). This is consistent with previous work^{2,4} and suggests that in this case the instability is of the Wigner-crystal type, with charge modulation both along the field and in the transverse directions.

In conclusion, we found that the range of the electron-electron interaction plays a crucial role both in the nature of a field-induced instability and in tunneling at energies above the corresponding gap. For a short-range interaction, there is a CDW instability which dominates tunneling in the whole energy interval. For a long-range interaction, depending on the depletion of the lowest Landau level, tunneling may exhibit two types of characteristic behavior. For not too strong depletion, the tunneling conductance decreases as a power-law of the relevant energy scale at higher energies and vanishes in a threshold-like manner near the gap (which for the long-range case is the Wigner-crystal gap). For a stronger depletion, the conductance first *increases* as a power of energy, then goes through a maximum and finally vanishes at the gap. These predictions are amenable to a direct experimental verification by tunneling into low-carrier-density materials, *e.g.*, graphite.

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